Frequency Estimation in Data Streams: Learning the Optimal Hashing Scheme

Dimitris Bertsimas and Vassilis Digalakis Jr.

Abstract—We present a novel approach for the problem of frequency estimation in data streams that is based on optimization and machine learning. Contrary to state-of-the-art streaming frequency estimation algorithms, which heavily rely on random hashing to maintain the frequency distribution of the data stream using limited storage, the proposed approach exploits an observed stream prefix to near-optimally hash elements and compress the target frequency distribution. We develop an exact mixed-integer linear optimization formulation, as well as an efficient block coordinate descent algorithm, that enable us to compute near-optimal hashing schemes for elements seen in the observed stream prefix; then, we use machine learning to hash unseen elements. We empirically evaluate the proposed approach on real-world search query data and show that it outperforms existing approaches by one to two orders of magnitude in terms of its average (per element) estimation error and by 45-90% in terms of its expected magnitude of estimation error.

Index Terms—Data streams, streaming frequency estimation, learning to hash, optimal hashing scheme.

1 INTRODUCTION

We consider a streaming model of computation [1], [2], where the input is represented as a finite sequence of elements from some finite universe (domain) which is not available for random access, but instead arrives dynamically and one at a time in a stream. We further assume that each element is identified by a unique key and is also associated with a set of features. One of the most fundamental problems in the streaming model is frequency estimation, i.e., given an input stream, estimate the frequency (number of occurrences) of each element. Notice that this can trivially be computed in space equal to the minimum of the universe and the stream size, by simply maintaining a counter for each element or by storing the entire stream, respectively. Nevertheless, data streams are typically characterized by large volume and, therefore, streaming frequency estimation algorithms should require small space, sublinear in both the universe and the stream size. Furthermore, streaming algorithms should generally be able to operate in a single pass (each element should be examined at most once in fixed arrival order) and in real-time (each element’s processing time must be low).

Example. Consider a stream of queries arriving on a server. The universe of all elements is the set of all possible queries (of bounded length) and each element is uniquely identified by the query text. Note that any unique query may appear multiple times in the stream. The features associated with a query could include, e.g., the query length, the unigram of the query text (possibly after some pre-processing), etc. Our goal is to estimate the frequency distribution of the queries, that is, the number of times each query appears in the stream, in space much smaller than the total number of unique queries.

Massive data streams appear in a variety of applications. For example, in search query monitoring, Google received more than 1.2 trillion queries in 2012 (which translates to 3.5 billion searches per day) [3]. In network traffic monitoring, AT&T collects over one terabyte of NetFlow [4] measurement data from its production network each day [2]. Moreover, the IPV6 protocol provides nearly $2^{128}$ addresses, making the universe of possible IP addresses gigantic, especially considering that, in many applications, we are interested in monitoring active IP network connections between pairs (source/destination) of IP addresses. Thus, being able to process a data stream in sublinear space is essential.

Maintaining the frequency distribution of a stream of elements is useful, not only as a sufficient statistic for various empirical measures and functionals (e.g., entropy [5]), but also to identify interesting patterns in the data. An example are the so-called “heavy-hitters” [6], that is, the elements that appear a big number of times, which, e.g., could be indicative of denial of service attacks in network traffic monitoring (see [2] for a detailed discussion of applications). Classical methods to address the heavy-hitter problem include the deterministic approach in [7], the sampling-based approach in [8], the approach in [9] that relies on group testing, and the sketching-based approach in [10].

In this paper, we address the problem of frequency estimation in data streams, under the additional assumption that a prefix of the input stream has been observed. Along the lines of [11], who address the same problem and extend classical streaming frequency estimation algorithms with a machine learning component, we aim to exploit the observed prefix and the features associated with each element, and develop data-driven streaming algorithms. The proposed algorithms satisfy the small-space requirement, as they significantly compress the input frequency vector,
and do operate in a single pass and in real-time, as their update and query times are constant (except for the training phase, which is more computationally demanding, since we perform optimization and machine learning).

1.1 Streaming Frequency Estimation Algorithms

A rich body of research has emerged in the streaming model of computation [1], [2]; the first streaming algorithms appeared in the early 1990s, to address, in limited space, problems such as finding the most frequently occurring elements in a stream [7]. A vast literature has since been developed, especially since the 1990s, and numerous problems, including complex machine learning tasks, such as decision tree induction [12], can now be solved in streaming settings.

Sketches [13] are among the most powerful tools to process streaming data. A sketch is a data structure which can be represented as a linear transform of the input. For example, in the context of frequency estimation, the input is the vector of frequencies (or frequency distribution) of the input elements and the sketch is computed by multiplying the frequency distribution by a fixed, “fat” matrix. Of course, for compactness, the matrix that performs the sketch transform is never explicitly materialized and is implicitly implemented via the use of random hash functions.

Any given sketch transform is defined for a particular task. Among the most popular sketching methods for the task of frequency estimation, are the Count-Min Sketch [10] and the Count Sketch [14], which both rely on random hashing and differ in their frequency estimation procedure. Historically, the so-called AMS Sketch [15], which addresses the task of estimating the sum of the squares of the frequencies of the input stream, was among the first sketching algorithms that have been proposed. Sketching algorithms have found numerous applications, including in measuring network traffic [16], in natural language processing [17], in signal processing and compressed sensing [18], and in feature selection [19].

1.2 Learning-Augmented Algorithms

The abundance of data that is available today has motivated the development of the field of learning-augmented algorithms, whereby traditional algorithms are modified to leverage useful patterns in their input data. More specifically, in the context of streaming algorithms, [20] and [21] augment with a machine learning oracle the Bloom filter [22], [23], a widely used probabilistic data structure that tests set membership, whereas [11] develop learning-based versions of the Count-Min Sketch and the Count Sketch.

Beyond streaming algorithms, [24] develop an optimal data-dependent hashing scheme for the approximate nearest neighbor problem, whereas [25] use machine-learned predictions to improve the performance of online algorithms. [26] use reinforcement learning and neural networks to learn workload-specific scheduling algorithms that, e.g., aim to minimize the average job completion time. Machine learning has also been used outside the field of algorithm design, e.g., in signal processing and, specifically, in the context of “structured” (instead of sparse) signal recovery [27] and in optimization. [28] and [29] propose machine learning-based approaches for variable branching in mixed-integer optimization, [30] use reinforcement learning to learn combinatorial optimization algorithms over graphs, [31] use interpretable machine learning methods to learn strategies behind the optimal solutions in continuous and mixed-integer convex optimization problems as a function of their key parameters, and [32] focus specifically on online mixed-integer optimization problems. Machine learning has also been popularized in the context of data management and, in particular, in tasks such as learning index structures [20] and query optimization [33], [34].

In this paper, we consider the same problem as in [11], namely learning-based streaming frequency estimation. However, contrary to [11], who combine a machine learning oracle with standard (conventional) streaming frequency estimation algorithms, such as the Count-Min Sketch, our approach does not rely on random hashing at all. Instead, we use optimization to learn an optimal (or near-optimal) hashing scheme from (training) data, and machine learning to hash “unseen elements,” which did not appear in the training data.

The proposed approach has connections with the field of learning to hash, a data-dependent hashing approach which aims to learn hash functions from a specific dataset (see [35] for a comprehensive survey). Learning to hash has mostly been considered in the context of nearest neighbor search, i.e., learning a hashing scheme so that the nearest neighbor search result in the hash coding space is as close as possible to the search result in the original space. Optimization-based learning to hash approaches include the works [36], [37], [38].

1.3 Contributions

Our key contributions can be summarized as follows:

- We develop a novel approach for the problem of frequency estimation in data streams that is based on optimization and machine learning. By exploiting an observed stream prefix, the proposed learning-based streaming frequency estimation algorithm achieves superior performance compared to conventional streaming frequency estimation algorithms.
- We present an exact mixed-integer linear optimization formulation, as well as an efficient block coordinate descent algorithm, that enable us to compute near-optimal hashing schemes and provide a smart alternative to oblivious random hashing schemes. This part of our work could be of independent interest, beyond the problem of frequency estimation in data streams.
- We apply the proposed approach to the problem of search query frequency estimation and evaluate it using real-world data. Our computational results indicate that the proposed approach notably outperforms state-of-the-art non-learning and learning-based approaches in terms of its estimation error. Moreover, the proposed approach is by construction interpretable and enables us to get additional insights into the problem of search query frequency estimation.
The rest of the paper is organized as follows. In Section 2, we formalize the streaming frequency estimation problem and present, at a high level, the Count-Min Sketch, the most widely used random hashing-based streaming frequency estimation algorithm, and the Learned Count-Min Sketch, a learning-augmented version of the Count-Min Sketch. Section 3 gives an overview of the proposed approach. In Section 4, we formulate the problem of learning the optimal hashing scheme using the observed stream prefix and develop an efficient optimization algorithm. In Section 5, we describe the frequency estimation procedure we apply, after the optimal hashing scheme is learned. Finally, Section 6 empirically evaluates the proposed approach on real-world search query data and Section 7 concludes the paper.

2 Preliminaries

In this section, we formally describe the problem of frequency estimation in data streams and present the state-of-the-art approaches to solving it.

Formally, we are given input data in the form of an ordered set of elements

\[ S = (u_1, u_2, \ldots, u_{|S|}) \]

where \( u_i \in U, \forall i \in [|S|] := \{1, \ldots, |S|\} \), and \( U \) is the universe of input elements. Each element \( u \in U \) is of the form

\[ u = (k, x) \]

where (without loss of generality) \( k \in [|U|] \) is a unique ID and \( x \in \mathcal{X} \) is a set of features associated with \( u \). The goal is, at the end of \( S \), given an element \( u \in U \), to output an estimate \( \hat{f}_u \) of the frequency

\[ \hat{f}_u = \sum_{i=1}^{|S|} \mathbf{1}(u_i = u) \]

of that element. We assume that both \( S \) and \( U \) are huge, so we wish to produce accurate estimates in space much smaller than \( \min\{|S|, |U|\} \). We work under the additional assumption that a prefix \( S_0 \) of the input stream has already been observed.

2.1 Conventional Approach: Random Sketches

The standard approach to attack this problem is the well-known Count-Min Sketch (CMS) [10], a probabilistic data structure based on random hashing that serves as the frequency table of \( S \). In short, CMS randomly hashes (via a random linear hash function \( \text{hash}(\cdot) \)) each element \( u \in U \) to a bucket in an array \( \mathcal{H} \) of size \( w \ll \min\{|S|, |U|\} \); whenever element \( u \) occurs in \( S \), the corresponding counter \( c_{\text{hash}(u)} \) is incremented. Since \( w \ll |U| \), multiple elements map to the same bucket and \( c_{\text{hash}(u)} \) overestimates \( f_u \). In practice, multiple arrays \( c^1, \ldots, c^d \) are maintained and the final estimate for \( f_u \) is

\[ \hat{f}_u = \min_{j \in [d]} c^j_{\text{hash}(u)}. \]

CMS provides probabilistic guarantees on the accuracy of its estimates, namely, for each \( u \in U \), with probability \( 1 - \delta \),

\[ |\hat{f}_u - f_u| \leq \epsilon \|f\|_1, \]

where \( \epsilon = \frac{\epsilon}{w} \) and \( \delta = e^{-d} \). In total, CMS consists of \( b = w \times d \) buckets.

2.2 Learning-Based Approach: Learned Sketches

To leverage the observed stream prefix, [11] augment the classical CMS algorithm as follows. Noticing that the elements that affect the estimation error the most are the so-called heavy-hitters (i.e., elements that appear many times), they propose to train a classifier

\[ h : \mathcal{X} \rightarrow \{\text{heavy, \overline{heavy}}\} \]

that predicts whether an element \( u = (k, x) \) is going to be a heavy-hitter or not. Then, they allocate \( b_{\text{heavy}} \) unique buckets to elements identified as heavy-hitters, and randomly allocate the remaining \( b_{\text{random}} = b - 2b_{\text{heavy}} \) buckets to the rest of the universe, using, e.g., the standard CMS. We call their algorithm the Learned Count-Min Sketch (LCMS).

An important remark is that each of the \( b_{\text{heavy}} \) unique buckets allocated to heavy-hitters should maintain both the frequency and the ID of the associated element. As explained, this can be achieved by using hashing with open addressing, whereby it suffices to store IDs hashed into \( \log b_{\text{heavy}} + t \) bits (instead of whole IDs which could be arbitrarily large) to ensure there is no collision with probability \( 1 - 2^{-t} \). Noticing that \( \log b_{\text{heavy}} + t \) is comparable to the number of bits per counter, the space for a unique bucket is twice the space of a normal bucket. The learning augmented algorithm is shown to outperform, both theoretically and empirically, its conventional, fully-random counterpart. Additionally, they prove that under certain distributional assumptions, allocating unique buckets to heavy-hitters is asymptotically optimal [11], [39]. In general, however, their approach remains heuristic, does not guarantee optimal performance, and possibly throws away information by taking hard, binary decisions.

3 Overview of the Proposed Approach

Motivated by the success of LCMS, we investigate an alternative, optimization-based approach in using the observed stream prefix to enhance the performance of the frequency estimator.

At a high level, the proposed two-phase approach works as follows. In the first phase, the elements that appeared in the stream prefix are optimally allocated to buckets based on their observed frequencies so that the frequency estimation error is minimized. Importantly, contrary to CMS-based approaches, our estimate for an element’s frequency is the average of the frequencies of all elements that are mapped to the same bucket. Therefore, we aim to assign “similar” elements to the same bucket. In the second phase, once we have an optimal allocation of the elements that appeared in the prefix to buckets, we train a classifier mapping elements to buckets based on their features. By doing so, we are able to provide estimates for unseen elements that did not appear in the prefix and hence their frequencies are not recorded.

The proposed hashing scheme consists of a hash table mapping IDs of elements that appeared in the prefix to buckets and the learned classifier. In addition, for each bucket, we need to maintain the sum of frequencies of all elements mapped therein. During stream processing, that is, once the estimator is ready, whenever an element that had appeared in the prefix re-appears, we increment the counter of the bucket to which the element was mapped. Finally,
to answer count-queries for any given element, we simply output the current average stored in the bucket where the element is mapped (either via the hash table or via the classifier).

4 Learning the Optimal Hashing Scheme

In what follows, we develop in full detail the proposed approach in learning the optimal hashing scheme.

4.1 Exact Formulation

Let \( S_0 = (u_1, ..., u_{T_0}) \) be the observed stream prefix. We denote by \( f_u^0 \) the empirical frequency of element \( u \) in \( S_0 \), i.e.,

\[
f_u^0 = \sum_{t=1}^{T_0} 1(u_t = u),
\]

and by \( f^0(S_0) \) the entire frequency distribution after observing \( S_0 \). Moreover, \( U_0 = \{u \in U : f_u^0 > 0\} \) is the set of all distinct elements that appeared in \( S_0 \) and let \( |U_0| = n \). We introduce \( n \times b \) binary variables, where \( b \) is the total number of available buckets, defined as

\[
z_{ij} = \begin{cases} 1, & \text{if } \text{ith element of } U_0 \text{ is mapped to bucket } j, \\ 0, & \text{otherwise}. \end{cases}
\]

Each row \( z_i \) of \( Z \) (where we denote \( |Z|_{ij} = z_{ij} \)) can be viewed as an one-hot binary hash code mapping element \( i \) to one of the buckets. At the end of the stream and given a fixed assignment for the variables \( z_{ij} \), our final estimate of the frequency of element \( i \) in \( U_0 \) is

\[
\tilde{f}_i = \sum_{j \in [b]} z_{ij} \left( \frac{\sum_{k \in [n]} z_{kj} f^0_k}{\sum_{k \in [n]} z_{kj}} \right).
\]

The resulting, e.g., absolute estimation error is \( \sum_{i \in [n]} |\tilde{f}_i - f^0_i| \); a natural objective is to pick the variables \( z_{ij} \) that minimize this absolute error in the observed stream prefix. An alternative objective we could pick is the expected magnitude of the absolute error \( \frac{1}{\sum_{k \in [n]} f_k} \sum_{i \in [n]} f_i \cdot |\tilde{f}_i - f^0_i| \), whereby it is assumed that the probability \( p_i \) of observing element \( i \) is equal to its empirical probability in the observed stream prefix, i.e., \( p_i := \frac{1}{\sum_{k \in [n]} f_k} \). In fact, this metric is used by \( [11] \) in their theoretical analysis. However, such an approach would heavily weigh the most frequently occurring elements and would probably produce highly inaccurate estimates for less frequent elements. As we would like to achieve a uniformly small estimation error, we stick to the former objective and select the variables \( z_{ij} \) that solve the following formulation.

\[
\begin{align*}
\min_{Z \in \{0,1\}^{n \times b}, E \in \mathcal{U}^{n \times b}} & \sum_{i \in [n]} \sum_{j \in [b]} z_{ij} \left( f^0_i - \frac{\sum_{k \in [n]} z_{kj} f^0_k}{\sum_{k \in [n]} z_{kj}} \right) \\
\text{s.t.} & \sum_{j \in [b]} z_{ij} = 1, \quad \forall i \in [n].
\end{align*}
\]

Problem \( 1 \) is a nonlinear binary optimization problem. As we show next, it can be as reformulated as a mixed integer linear optimization problem by introducing auxiliary variables, new constraints, and a big-M constant:

\[
\begin{align*}
\min_{Z \in \{0,1\}^{n \times b}, E \in \mathcal{U}^{n \times b}, \Theta \in \mathbb{R}^+} & \sum_{i \in [n]} \sum_{j \in [b]} \theta_{ij} \\
\text{s.t.} & \sum_{k \in [n]} \theta_{ikj} - f^0_i \sum_{k \in [n]} z_{kj} + \sum_{k \in [n]} f^0_k z_{kj} \geq 0, \\
& \sum_{k \in [n]} \theta_{ikj} + f^0_i \sum_{k \in [n]} z_{kj} - \sum_{k \in [n]} f^0_k z_{kj} \geq 0, \\
& \theta_{ikj} \geq e_{ij} - M(1 - z_{kj}), \\
& \theta_{ikj} \leq e_{ij}, \\
& \theta_{ikj} \leq M \sum_{k \in [n]} z_{kj}, \\
& \sum_{j \in [b]} z_{ij} = 1, \\
& \forall i \in [n].
\end{align*}
\]

Problem \( 2 \) consists of \( \mathcal{O}(n^2 b) \) variables and constraints. Solving a mixed integer linear optimization problem of that size can still be prohibitive in the applications we consider. For example, in our experiments, we map up to tens of thousands of elements to up to thousands of buckets, so Formulation \( 2 \) would consist of variables and constraints in the order of \( 10^{11} \). Therefore, we next develop a tailored block coordinate descent algorithm that works well in practice.

4.2 Efficient Block Coordinate Descent Algorithm

By exploiting the problem structure, we propose the following efficient block coordinate descent algorithm (Algorithm 1) that can be used to either heuristically solve Formulation \( 1 \) or compute high-quality warm starts for Formulation \( 2 \). In each iteration, Algorithm 1 examines sequentially and in random order all \( n \) blocks of \( b \) variables \( z_{ij}, \ i \in [n] \). Notice that each block contains all possible mappings of a particular element to any bucket. For each element \( i \), we greedily select the mapping that minimizes the overall estimation error. To do so, we remove element \( i \) from its current bucket and compute the estimation error associated with each bucket \( j \), first with element \( i \) allocated to bucket \( j \) and then without element \( i \). We allocate element \( i \) to the bucket \( j^* \) that minimizes the sum of all error terms.

Concerning the algorithm’s initialization, we start from a random allocation of elements to buckets. Alternatively, we could sort elements in \( U_0 \) in terms of their observed frequencies and allocate the first \( \left\lfloor \frac{\left| U_0 \right|}{b} \right\rfloor \) elements to the first bucket, the next \( \left\lfloor \frac{\left| U_0 \right|}{b} \right\rfloor \) to the second bucket, and so forth, or we could even use the heavy-hitter heuristic (that is, assign heavy hitters to their own bucket and the remaining elements at random). The algorithm terminates when the
Algorithm 1 Block Coordinate Descent Algorithm.

Input: Observed frequency vector \( f^0 \in \mathbb{N}^n \), number of buckets \( b \in \mathbb{N} \).

Output: Learned one-hot hashing scheme \( Z \in \{0,1\}^{n \times b} \).

Initialize \( Z \) satisfying \( \sum_{i \in [n]} z_{ij} = 1, \forall i \in [n] \).
\[
\varepsilon_0 \leftarrow \sum_{i \in [n]} \sum_{j \in [b]} z_{ij} \left| f_i^0 - \frac{\sum_{k \in [n]} z_{kj} f_k^0}{\sum_{k \in [n]} z_{kj}} \right|
\]
\( t \leftarrow 0 \)
repeat
\( t \leftarrow t + 1 \)
\begin{align*}
\varepsilon_t &\leftarrow \sum_{i \in [n]} \sum_{j \in [b]} z_{ij} \left| f_i^0 - \frac{\sum_{k \in [n]} z_{kj} f_k^0}{\sum_{k \in [n]} z_{kj}} \right| \\
\end{align*}
until \( \varepsilon_{t-1} - \varepsilon_t < \epsilon \)
return \( Z \)

improvement in estimation error is negligible; in case we are willing to obtain an intermediate solution faster, the termination criterion can be set to a user-specified maximum number of iterations. Given that algorithm is not guaranteed to converge to a globally optimum solution, the process can be repeated multiple times from different starting points.

Algorithm 1 can be efficiently implemented so that the complexity of each iteration is \( O(nb^2) \) and, in practice, it converges to a local optimum after a few tens of iterations. As we empirically show, it produces high-quality solutions.

4.3 Extended Exact Formulation

We next extend Formulation (1) to take the features associated with each element into account when computing the optimal mapping of elements to buckets. For \( \lambda \in [0, 1] \), we have

\[
\min_{Z \in \{0,1\}^{n \times b}} \sum_{i \in [n]} \sum_{j \in [b]} z_{ij} \left[ \lambda \left| f_i^0 - \frac{\sum_{k \in [n]} z_{kj} f_k^0}{\sum_{k \in [n]} z_{kj}} \right| \right. \\
\left. + (1 - \lambda) \sum_{k \in [n]} z_{kj} \| x_i - x_k \|^2 \right]
\]

s.t. \( \sum_{j \in [b]} z_{ij} = 1, \forall i \in [n] \).

The parameter \( \lambda \in [0, 1] \) controls the trade-off between hashing schemes that map to the same bucket elements that are similar in terms of their observed frequencies in the prefix \( \lambda \rightarrow 0 \) and hashing schemes that put more weight on the elements’ feature-wise similarity \( \lambda \rightarrow 1 \). Following a similar procedure as with Formulation (1), Formulation (3) can also be linearized. Moreover, it can be solved via efficient heuristic methods, such as a modified version of Algorithm 1.

5 Frequency Estimation

In this section, we describe the frequency estimation component of the proposed estimator, which, in its simplest form, consists of a multi-class classifier.

5.1 Frequency Estimation for Elements Seen in the Prefix

Once the optimal assignment \( Z \) is computed, we essentially have a hash code \( h_i = \sum_{j \in [b]} z_{ij} \cdot j \cdot (z_{ij} = 1), i \in [n] \), for each element \( u \in \mathcal{U}_0 \). Therefore, for elements \( u \in \mathcal{U}_0 \), we simply estimate their frequency as

\[
\hat{f}_u = \frac{\sum_{k \in [n]: h_k = h_i} f_k}{\sum_{k \in [n]: h_k = h_i} 1}.
\]

5.2 Similarity-Based Frequency Estimation for Unseen Elements

To be able to produce frequency estimates for elements that did not appear in the prefix, i.e., \( u \in \mathcal{U} \setminus \mathcal{U}_0 \), we formulate a multi-class classification problem, mapping elements to buckets based on their features. Formally, we search for a function

\[
g: \mathcal{X} \rightarrow [b].
\]

The training set consists of all data points in

\[
\{(x_i, h_i) : u_i = (k_i, x_i) \in \mathcal{U}_0\},
\]

that is, all feature-hash code tuples for elements that appeared in the prefix. Such a classifier will allow us to
estimate the frequencies of unseen elements based on the average of the frequencies of elements that “look” similar. Our estimate for element \( u = (k, x) \in \mathcal{U} \) is then

\[
\tilde{f}_u = \frac{k \sum_{k \in \mathcal{U}, h_k = g(x)} f_k}{\sum_{k \in \mathcal{U}, h_k = g(x)} 1}.
\]

### 5.3 Adaptive Counting

So far, we have described a static approach; we learn the optimal hashing scheme for the elements that appear in the stream prefix and then keep track only of their frequencies. Our estimated frequencies for all elements are based only on the frequencies of elements in \( \mathcal{U}_0 \) (which appeared in \( S_0 \)). We next describe a dynamic approach, which keeps track of the frequencies of elements beyond the ones in \( \mathcal{U}_0 \). At a high level, the adaptive approach is based on approximately counting the distinct elements in each bucket. We work as follows.

1) We learn the optimal hashing scheme based on the observed stream prefix and train a classifier mapping elements to buckets, as outlined above. For each bucket, we only record the number of elements that map therein (instead of storing the IDs of the elements that map to this bucket). We use the classifier to determine which bucket any element is mapped to.

2) For each bucket, we maintain a Bloom filter \( BF \), i.e., a probabilistic data structure that, given a universe of elements \( \mathcal{U} \) and a set \( \mathcal{U}' \subseteq \mathcal{U} \), probabilistically tests, for any element \( u \in \mathcal{U} \), whether \( u \in \mathcal{U}' \). If \( u \in \mathcal{U}' \), then we deterministically have that \( BF(u) = 1 \). However, if \( u \notin \mathcal{U}' \), then it need not be the case that \( BF(u) = 0 \) (therefore a Bloom filter is prone to false positives).

3) We initialize the Bloom filter based on the elements \( u \in \mathcal{U}_0 \). Therefore, all elements \( u \in \mathcal{U}_0 \) will initially have \( BF(u) = 1 \). On the other hand, elements \( u \notin \mathcal{U}_0 \) may initially have either \( BF(u) = 0 \) or \( BF(u) = 1 \).

4) For every subsequent element \( u \) that appears in the stream after the stream prefix \( S_0 \) has been processed, we map it to a bucket \( b \) using the trained classifier. Then, we test whether we have already seen \( u \) using the bucket’s Bloom filter. If \( BF(u) = 0 \), we increase both the frequency \( f_b \) and the number of elements \( c_b \) in the bucket \( b \), and we set \( BF(u) = 1 \). If \( BF(u) = 1 \), we only increase the frequency in \( b \).

5) When queried for the frequency of any element \( u \in \mathcal{U} \), regardless of whether it appeared in \( \mathcal{U}_0 \) or not, we estimate

\[
\tilde{f}_u = \frac{f_b}{c_b},
\]

where \( b \) is the bucket in which \( u \) is mapped using the classifier.

### 6 Experimental Evaluation: Search Query Estimation

In this section, we empirically evaluate the proposed approach on real-world search query data. The task of search query frequency estimation seems particularly suited for the proposed learning-based approach, given that popular search queries tend to appear consistently across multiple days.

#### 6.1 Dataset

In the lines of \([11]\), we use the AOL query log dataset, which consists of 21 million search queries (with 3.8 million unique ones) collected from 650 thousand anonymized users over 90 days in 2006. Each query is a search phrase in free text; for example, the 1st most common query is “google” and appears 251,463 times over the entire 90-day period, the 10th is “www.yahoo.com” and its frequency is 37,436, the 100th is “mys” and its frequency is 5,237, the 1000th is “sharon stone” and its frequency is 926, the 10,000th is “online casino” and its frequency is 146, and so forth. As shown in \([11]\), the distribution of search query frequency indeed follows the Zipfian law and hence the setting seems ideal for their proposed algorithm (LCMS).

#### 6.2 Baselines

As baselines, we use CMS (the standard Count-Min Sketch, referred to as count-min) and LCMS (the learned Count-Min Sketch with the heavy-hitter heuristic, referred to as heavy-hitter). For each method, we maintain multiple versions corresponding to different values of the method’s hyperparameters and report the best performing version. More specifically, for fixed sketch size (i.e., total number of buckets \( b \)), we report the best performing for count-min’s depth from the set \( \{1, 2, 4, 6\} \) and for heavy-hitter’s depth \( d \in \{1, 2, 4, 6\} \) and number of heavy-hitter buckets \( b_{\text{heavy}} \in \{10, 10^2, 10^3, 10^4\} \) (provided that \( b_{\text{heavy}} \) fits within the available memory, i.e., \( b_{\text{heavy}} \leq \lambda/2 \)). Additionally, we assume that heavy-hitter has access to an ideal heavy-hitter oracle, i.e., the IDs of the heavy-hitters in the test set (over the entire 90-day period) are known. Therefore, we compare the proposed method with the ideal version of the method proposed in \([11]\), which was in fact shown to significantly outperform any realistically implementable version of heavy-hitter that relied upon non-ideal heavy-hitter oracles (e.g., recurrent neural network classifier).

#### 6.3 Remarks on the Learned Hashing Scheme

As far as the proposed method (referred to as opt-hash) is concerned, we make the following remarks:

- We consider the first day to be the observed stream prefix \( S_0 \) and use (part of) the queries \( u \in \mathcal{U}_0 \subseteq \mathcal{U}_0 \) therein (along with their number of occurrences during the first day) to learn the optimal hashing scheme via Algorithm \([1]\).
- The first day consists of over 200,000 unique queries and just storing their IDs would require 200,000 buckets. Thus, we randomly sample a subset of the observed queries, with probabilities proportional to their observed frequencies. We use the sampled subset of queries as input to Algorithm \([1]\).
- For fixed number of total buckets \( b_{\text{total}} \), we need to determine the ratio \( c \) between the number of buckets \( b \) that the learned hashing scheme will consist of and
the number of queries $n$ whose IDs we will store. Therefore, for user-specified $b_{total}$ and $c$, we pick $b$ and $n$ according to

$$n = \frac{b_{total}}{1 + c}, \quad b = b_{total} - n.$$  

In our experiments, we use validation data to select $c \in \{0.03, 0.3\}$.  

- For the classifier $g : \mathcal{X} \rightarrow [b]$, mapping unseen queries $u \in \mathcal{U} \setminus \mathcal{U}_t$ to buckets (as per Section 4.1), we explore various tree-based models, including CART [40], random forest [41], and XGBoost [42]. We found that random forest achieves the best trade-off between training time and classification accuracy and use this model in the results we report.  

- To create input features for the classifier $g$, we follow a simple bag-of-words approach and only keep the 500 most common words in the training queries. We also include as features the number of ASCII characters in the query text, the number of punctuation marks, the number of dots, and the number of whitespaces. As a result, the proposed approach is simple and interpretable, yet strong (as we show next).

6.4 Results

We implement our experiments in Python 3.6 and use the Scikit-learn machine learning package [43]. We independently repeat each experiment 5 times and report the averaged error, as well as its standard deviation. We remark that each bucket consumes 4 bytes of memory and hence the total number of buckets used in each experiment can be calculated as $b = \frac{m \cdot 1024}{4}$, where $m$ is the size of the estimator in KB. Moreover, we denote by $\mathcal{U}_t$ the set of queries that appear in day $t$, and by $f_u^t$ and $\tilde{f}_u^t$ their aggregated true frequencies and estimated frequencies, respectively, between days $0$ and $t$.

In Figure 1, we show the estimation error as function of the estimator’s size in KB, after the $30^{th}$ and the $70^{th}$ day. On the the left (Figures 1a and 1c), we plot the average (per element) estimation error

$$\frac{1}{|\mathcal{U}_t|} \sum_{u \in \mathcal{U}_t} |f_u^t - \tilde{f}_u^t|.$$  

On the the right (Figures 1b and 1d), we plot the expected magnitude of the absolute estimation error

$$\frac{1}{|\mathcal{U}_t|} \sum_{u \in \mathcal{U}_t} f_u^t \cdot |f_u^t - \tilde{f}_u^t|.$$  

Notice that the former metric is expressed in a per element scale, that is, we normalize the overall error by the total number of elements $|\mathcal{U}_t|$ and hence all elements are penalized uniformly, whereas the second metric, the expected magnitude of the absolute estimation error, penalizes elements proportionally to their actual frequencies, as per Section 4.1.

We observe that the trend in the estimation error is very similar after the $30^{th}$ and the $70^{th}$ day. What changes
is the absolute value of the estimation error, which, as expected, deteriorates with time, uniformly for all methods. The proposed method opt-hash consistently outperforms its competitors, in terms of both metrics. Unsurprisingly, as the size of all estimators increases, their errors drop. This is the case with both the average and the expected estimation error. We make the following additional remarks:

- The superiority of opt-hash is most notable in terms of average (per element) error. This is partly due to the fact that opt-hash does a substantially better job at estimating the frequencies of rarely occurring queries. In particular, queries that appear very few times are placed in the same bucket and hence the estimation error on them is small. In contrast, heavy-hitter and count-min often place such queries in the same bucket with queries of medium or even high frequencies, which produces big estimation error.

- The expected magnitude of the estimation error of heavy-hitter and count-min does seem to slowly converge towards that of opt-hash when the estimators’ size becomes sufficiently large. This indicates that opt-hash is particularly suited for low-space regimes and can achieve much more effective compression of the frequency vector.

- As far as heavy-hitter and count-min are concerned, the former does produce better estimates, which is in agreement with the results in [11]. The improvement is much more notable in terms of the expected magnitude of the estimation error. This observation is to be expected as well, given that heavy-hitter makes zero error on the most frequently occurring elements, which are heavily weighed in this metric.

Figure 2 reports the estimation error as function of time (in days), for two different memory configurations (4 KB in Figures 2a and 2b, 120 KB in Figures 2c and 2d). The superiority of opt-hash is preserved over time, in terms of both metrics. Moreover, we observe opt-hash achieves the smallest standard deviation in its estimation error. This can be attributed to the fact that the mappings of elements to buckets are more stable than those of heavy-hitter and count-min, as they are obtained via optimization instead of randomization; the main source of randomness for opt-hash is the classifier.

We next experiment with memory configurations that vary between 1.2 KB and 120 KB, and compare opt-hash with count-min and heavy-hitter. The proposed approach provides an average improvement (over the entire 90-day period) by one to two orders of magnitude, in terms of its average (per element) absolute estimation error, and by 45-90%, in terms of its expected magnitude of estimation error. For example, with 120 KB of memory, opt-hash makes an average absolute estimation error of $\sim 29$ in estimating the frequency of each query, whereas the error of heavy-hitter is $\sim 479$ (Figure 1a). With 4 KB of memory, the errors of opt-hash and heavy-hitter are $\sim 167$ and $\sim 14,661$, respectively (Figure 1c).
An additional feature of opt-hash is that, by using interpretable features in its machine learning component, it provides insights into the underlying frequency estimation problem. In particular, the features that were consistently marked as most important are the four counts (i.e., number of ASCII characters in the query text, the number of punctuation marks, the number of dots, and the number of whitespaces), as well as the words “com,” “www,” “google,” and “yahoo.” Intuitively, this observation makes sense. For instance, a large number of ASCII characters and whitespaces would be indicative of a big query with multiple words, making it more likely to be rare. On the other hand, a query containing the word “google” would be more likely to be common, given that “google” is consistently part of the most frequently occurring queries.

7 Conclusion
In this paper, we developed a novel approach for the problem of frequency estimation in data streams that relies on the use of optimization and machine learning on an observed stream prefix. First, we formulated and efficiently solved the problem of optimally (or near-optimally) hashing the elements seen in the prefix to buckets, hence providing a smart alternative to oblivious random hashing schemes. Next, we trained a classifier mapping unseen elements to buckets. As we discussed, during stream processing, we only keep track of the frequencies of those elements that appeared in the prefix; our estimate the frequency of any element (either seen or unseen) is the average of the frequencies of all elements that map to the same bucket. We also described an adaptive approach that enables us to update our compressed frequency vector and keep track of the frequencies of all elements. We applied the proposed approach to the problem of search query frequency estimation and evaluated it using real-world data and empirically showed that the proposed learning-based streaming frequency estimation algorithm achieves superior performance compared to existing streaming frequency estimation algorithms.

Acknowledgments
We would like to thank Ryan Cory-Wright for his fruitful comments.

References


Dimitris Bertsimas is the Associate Dean of Business Analytics, Boeing Professor of Operations Research and faculty director of the Master of Business Analytics at MIT. He received his SM and PhD in Applied Mathematics and Operations Research from MIT in 1987 and 1988 respectively. He has been MIT faculty since 1988. His research interests include optimization, machine learning, and applied probability, and their applications in health care, finance, operations management, and transportation. He has co-authored more than 200 scientific papers and five graduate level textbooks and has received numerous awards, with the most recent being the John von Neumann Theory Prize, INFORMS, and the President’s award, INFORMS, both in 2019.

Vassilis Digalakis Jr. is a PhD candidate at MIT’s Operations Research Center, advised by Prof. Dimitris Bertsimas. Prior to joining MIT, he earned his Diploma in Electrical and Computer Engineering from the Technical University of Crete, Greece, in 2018. His research interests lie at the intersection of machine learning and optimization, with application to big-data settings.